Nodal Angles Method

The Angles solution method for nodal dispatch is theoretically equivalent to the Shift Factor method. Both methods model the DC Powerflow approximation. The DC Powerflow is used within Aurora's Optimal Power Flow module to solve for the most optimal dispatch of generation to meet the demand subject to system constraints. Although, the two methods are theoretically equivalent, they vary significantly in their implementation.

The Angles solution method explicitly models Kirchoff's Law at every bus in the network. Kirchoff's Law is a fundamental physical law that governs the distribution of power in a transmission network. The law states that "Net power injection at every bus must be zero." In other words, for every bus in the network,

$$Generation - Demand = \text{Net Power Flow}$$

where, $Generation$ represents power generation, $Demand$ represents demand at the bus, and Net Power Flow represents the net of incoming and outgoing power flowing over all transmission lines connected with the bus.

The DC Powerflow paradigm assigns an imaginary quantity to every bus. This imaginary quantity is commonly referred to as the bus's *angle* and represented as θ_i for the i^{th} bus. The power flowing from Bus i to Bus j over a transmission line connecting the two buses is then given

$$P_{i \to j} = B_{ij}(\theta_i - \theta_j)$$

where B_{ij} is the transmission line's admittance.

Collecting the equations in a matrix form, the Optimal Power Flow (N-0) formulation is given by:

$$\begin{aligned} \underset{g_1, g_2, \ldots}{\text{minimize}} \quad & \mathcal{C}(g_1, g_2, \ldots) \\ \text{subject to:} \quad & Y_{bus}\theta = G - D \\ & g_{\min} \leq g_i \leq g_{\max} \ \forall i \\ & Br(i,j)_{\min} \leq B_{ij}(\theta_i - \theta_j) \leq Br(i,j)_{\max} \ \forall i,j \end{aligned}$$

where, G and D are the vectors of generation and demand at every bus, while θ is the vector of bus angles, and $\mathcal{C}(g_1, g_2, \ldots)$ represents the system-wide cost of generation. The equality condition specifies the power distribution through the transmission network given the vector of bus injections. The first set of inequality constraints represents the limits of the generation at a particular bus, while the final constraints specify the transmission limits. Y_{bus} is commonly referred is to as the *admittance* matrix.

For N-x SCOPF, the Angles solution method uses Line Outage Distribution Factor (LODFs) in an analogous way to the Shift Factor method.